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INVESTIGATION OF MATHEMATICAL PROBLEMS OF ROLLING THE MELTED METAL BETWEEN THE COOLING ROLLERS

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For practical solution of the problem of determination of dynamics of the smelt solidification in time and prediction of the metal structure (including the calculation and analysis of temperature fields, as well as determination of velocity of the solidification front in time) a mathematical model was developed. The thermal conductivity equation was supposed to be at the basis of this model. Continuous process of pulling the metal layer can be represented as flows of viscous incompressible layer between two elastic-plastic surfaces (rollers), moving with a certain velocity.

Keywords: solidification, mathematical modeling, liquid phases, metal structure, continuous process, rolling.

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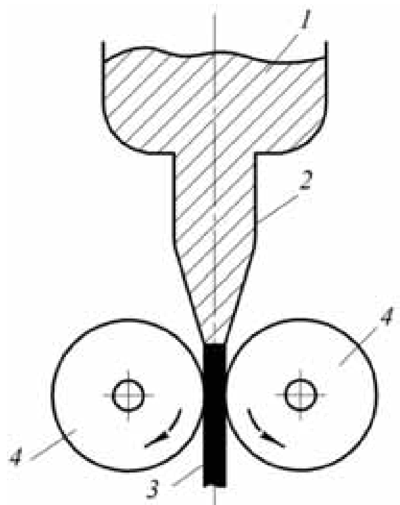
Practical problem of determining the dynamics of solidification of the smelt in time and prediction of metal structure includes the calculation and analysis of temperature fields, determination of the velocity of the solidification front in time, can be solved by means of mathematical modeling.In order to solve this problem the mathematical model was developed in the basis of which there was supposed to be thermal conductivity equation. Formation of a continuous layer of metal is a complex irreversible process, consisting of a series of simple phenomena, which in this case cannot be considered out of interaction with each other. Irreversibility of the process is associated primarily with the irreversibility of heat and mass transfer, the internal motion in the solid and liquid phases. In the general case a quantitative description of the process is based on the consideration of non-equilibrium thermodynamics of uninsulated heterogeneous system consisting of few components, separatedphases and the environment bounding hull.

Continuous process of pulling the metal layer can be represented as a flow of a viscous incompressible layer between two elastic-plastic surfaces (rollers), moving with a certain velocity (Figure 1). Moreover, an arbitrarily selected point in the cooling layer is characterized by a continuous change in temperature, pressure (stress), speed, distance to the boundary of the transition from liquid to solid state.

Thus, consideration of the cast layer 3 is bounded on both sides by curved surfaces (rollers) 4. Layer 3 - viscous incompressible fluid.



1 - liquid metal smelt; 2 - form for the metal; 3 - cooled metal layer; 4 - cooled rollers.
Figure 1 - Pulling process scheme related to a continuous layer of metal

Formalization of the mathematical model

Since the problem of the motion of continuously cast layer is closely related to the problem of heat exchange, the key is the thermal conductivity equation in a general form for a moving continuous medium (layer 3) in which there are distributed sources of heat of phase transition, depending on the specific heat of phase transition [1]:

$$\rho \cdot \frac{\partial}{\partial t}\left[c(T)T - L\Psi\right] = = div\left[\lambda(T) gradT\right] + Q_v \quad (1)$$

and Fourier thermal conductivity law $q = -\lambda(T) gradT$. Hereρ ρ - density; c - thermal capacity; λ - thermal conductivity; T - temperature; L - specific heat of phase transition (solidification); ψ - proportion of solid phase (the liquid phase $\psi = 0$, the solid phase $\psi = 1$, in the mushy zone $0 < \psi < 1$); Qv $(x, y, z, t)$is thefunction characterizing the mass allocated during the heat solidification.

To obtain a unique solution of the problem it is necessary to supplement its initial and boundary conditions [2-3]:

$$\lambda_m \frac{\partial T}{\partial x} = 0 \quad (2)$$

$$\lambda_m \frac{\partial T}{\partial x} = \alpha(T_m - T_a) \quad (3)$$

where T_m – metal temperature, °C
T_a – air temperature, °C

$$T(x,0) = 1300\,^\circ C \quad x = \frac{H}{2} \quad (4)$$

The phase boundary “solid metal - liquid melt” is given by the Stefan boundary condition:

$$\lambda_1 \frac{\partial T_1(t,\xi(t))}{\partial x} - \lambda_2 \frac{\partial T_2(t,\xi(t))}{\partial x} = = L\frac{d\xi(t)}{dt}, \quad (5)$$

where $\xi(t)$ – equation of the curve separating phases “solid metal - liquid melt”, L – heat change of state, J/K (empirically determined value for the transition of the liquid melt into the solid metal), x – normal to the curve, $T_1(t,x)$ – temperature of the solid phase (solid metal), $T_2(t,x)$ – temperature of the liquid phase (liquid melt), λ_1 – temperature diffusivity coefficient of the solid metal, λ_2 – temperature diffusivity coefficient of the liquid melt.

Let’s define the shape of the curve $\xi(t)$. We seek a solution to the thermal conductivity equation (1) in the following automodel form:

$$T(t,x) = f(z), \text{ where } z = \frac{r}{\sqrt{t}} \quad (6)$$

Substituting (6) into (1) we come to the following ordinary differential equation:

$$-\frac{1}{2} f'(z)\cdot z = \lambda\left(f''(z) + \frac{1}{z} f'(z)\right) \quad (7)$$

From which:

$$f(z) = C_1 \int \frac{\exp\left(-\frac{z^2}{4\lambda}\right)}{z} dz + C_2 \quad (8)$$

where C_1 and C_2 – arbitrary constants of integration.

After determining the shape of the curve *ξ (t)*, we substitute (6) into the Stefan boundary condition (5). We get

$$\lambda_1 \frac{1}{\sqrt{t}} f_1'\left(\frac{\xi(t)}{\sqrt{t}}\right) - \lambda_2 \frac{1}{\sqrt{t}} f_2'\left(\frac{\xi(t)}{\sqrt{t}}\right) = L\frac{d\xi}{dt},$$

from which

$$\frac{\xi(t)}{\sqrt{t}} = \alpha = const,$$

$$\lambda_1 f_1'(\alpha) - \lambda_2 f_2'(\alpha) = \frac{L}{2}\alpha \quad (9)$$

Consequently

$$\xi(t) = \alpha\sqrt{t} \quad (10)$$

Where the coefficient α is defined as the solution of the transcendental equation (9) with the known value of *L* aggregate heat transition of molten liquid into the solid state.

Knowing the equation of the curve *ξ (t)*, separating the phase"liquid smelt - solid metal", we can reduce the solution of the original problem to the solution of thethermal conductivity equation with generalized (discontinuous) temperature conductivity coefficient:

$$\frac{\partial T}{\partial t} = \lambda(t,x)\left(\frac{\partial^2 T}{\partial x^2}\right),$$

where

$$\lambda(t,x) = \begin{cases} \lambda_1, if\ 0 \le x < \theta R + \xi(t) \\ \lambda_2, if\ \theta R + \xi(t) \le x < R \end{cases}$$

Initial conditions

$$T\big|_{t=0} = T_{s.m}\ at\ 0 \le x < \theta R + \xi(t).$$

$$T\big|_{t=0} = T_{l.m}\ at\ \theta R + \xi(t) \le x < R$$

Fig.2-3 shows graphs characterizing the dependence of the degree of amorphization of parameters of the pulling process (rapid casting) related to a metal layer between the cooling rolls.

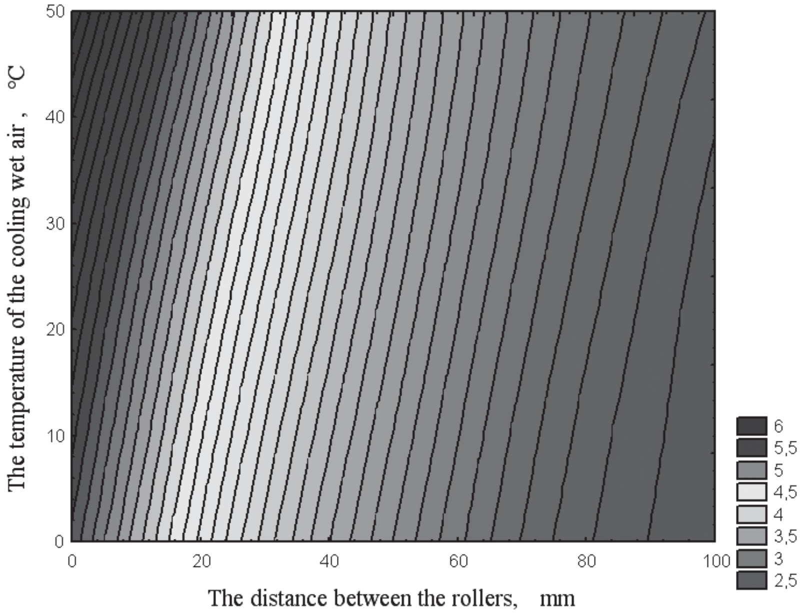


Fig. 2. Graph of dependence of the degree of amorphizationon the cooling wet air temperature and the distance between the rollers

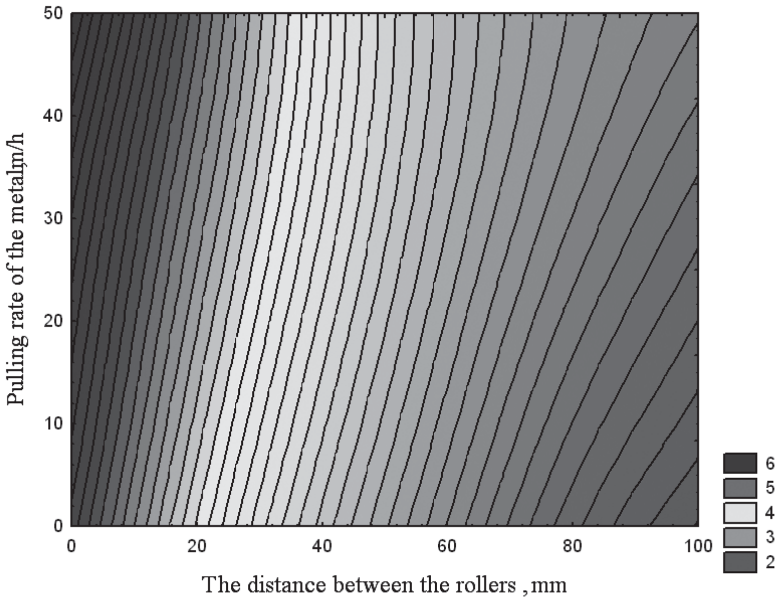


Fig. 3. Graph of dependence of the degree of amorphizationon themetalpulling rateand the distance between the rollers

Conclusions:

1. Authors have practically proved that under certain modes of the processof pulling (rapid casting) of metal layer between the cooling rolls it is possible to obtain an amorphous structure at the layer boundaries.

2. Obtaining an amorphous structure of the metal pullingprocess (fast casting) is possible only in terms of interaction of such mode parameters (control factors) as the distance between the rollers, metalpulling speed and the cooling wet airtemperature. In order to increase the degree of amorphization of the casting process it is necessary to increase the temperature of the cooling wet air and the smeltpulling speed. But the main parameter of the process that has the greatest impact on the degree of metalamorphization is the distance between the rollers.At the minimum values of distance degree of amorphization has a maximum value.

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